Alg. Lesson 3-5 Arithmetic Sequences as Linear Functions

OAS: : A1.A.3.1 Solve equations involving several variables for one variable in terms of the others.
A1.A.3.4 Evaluate linear, absolute value, rational, and radical expressions. Include applying a nonstandard operation such as a@ b = 2a + b.
A1.A.3.5 Recognize that arithmetic sequences are linear using equations, tables, graphs, and verbal descriptions. Use the pattern, find the next term.
A1.A.4.1 Calculate and interpret slope and the x- and y-intercepts of a line using a graph, an equation, two points, or a set of data points to solve realworld and mathematical problems
A1.F.1.2 Identify the dependent and independent variables as well as the domain and range given a function, equation, or graph. Identify restrictions on the domain and range in real-world contexts.
A1.F.1.3 Write linear functions, using function notation, to model real-world and mathematical situations.
A1.F.2.1 Distinguish between linear and nonlinear (including exponential) functions arising from real-world and mathematical situations that are represented in tables, graphs, and equations. Understand that linear functions grow by equal intervals and that exponential functions grow by equal factors over equal intervals.
A1.F.2.2 Recognize the graph of the functions F(x) = x and f(x) = |x| and predict the effects of transformations [f(x + c) and f(x) + c, where x is a positive or negative constant] algebraically and graphically using various methods and tools that may include graphing calculators.
A1.F.3.1 Identify and generate equivalent representations of linear equations, graphs, tables, and real-world situations.
A1.F.3.2 Use function notation; evaluate a function, including nonlinear, at a given point in its domain algebraically and graphically. Interpret the results in terms of real-world and mathematical problems.

<u>Recognizing Arithmetic Sequences</u>

A sequence is simply a pattern of numbers in a specific order. An arithmetic sequence is has a constant difference between successive terms.

Ex) The following are times and distances for a rowing team. Because the difference of successive terms is constant, it is an arithmetic sequence, which means it is also a linear function.

Distance (m)	400	800	1200	1600	2000
Time (min : sec)	1:32	3:04	4:36	6:08	7:40

+ 1:32 + 1:32 + 1:32 + 1:32

<u>Identify Arithmetic Sequences</u>

Remember that arithmetic sequences are linear functions, so if we can determine that a sequence is arithmetic, then we know it is a linear function!

Ex) Determine whether each sequence is arithmetic. Explain.

A) -4, -2, 0, 2… (the three periods are called an ellipsis. This means that there are more terms in the sequence that are not listed.)

The sequence increases by two each time, so this IS an arithmetic sequence, so it is also a linear function.

B) 1, 4, 9, 25… The sequence increases by 3, then 5, then 16. The differences are not constant, therefore this is not an arithmetic sequence.

<u>Find the next term</u>

If we continue a sequence with the constant difference, we can find the next term.

Ex) 15, 9, 3, -3 Find the next term

The constant difference is -6, so we continue this sequence and find the next term, which would be -3 + -6, or -9.

Ex) 9.5, 11, 12.5, 14 Find the next term.

The constant difference is +1.5, so we add 1.5 to 14 and the next term is 15.5.

Find the "n"th term.

The "n"th term, just simply means a number in the sequence that we don't know where it sits in the sequence.

For example: in this situation, "A" stands for the terms, and "d" stands for the difference in the sequence.

First term--8	8	A_1	
Second Term--11	8 + 1(3)	A_2	$A_1 + d$
Third Term--14	8 + 2(3)	A_3	$A_1 + 2(d)$
Fourth Term--17	8 + 3(3)	A_4	$A_1 + 3(d)$
N th Term-- ??	8 + (n – 1)(3)	A_n	$A_n + (n - 1)(d)$

This also shows the equation for the nth term of a sequence:

$$a_n = a_1 + (n - 1)d,$$

Where a_1 is the first term in the sequence and d is the constant difference in the terms.

Ex) What is the 9th term of the sequence? -12, -8, -4, 0

Use the formula: $a_n = a_1 + (n-1)d$
A_1 is the first term in the sequence, which in this case is -12.
N is the nth term. In this problem, they are asking for the 9th term, so n = 9.
D is the constant difference. This sequence is increasing by 4 each time, so d = 4.

$$a_n = -12 + (9-1)(4)$$

We use order of operations to solve:

$a_n = -12 + (9-1)(4)$

$$a_n = -12 + (8)(4)$$

$$a_n = -12 + 32$$

$$a_n = 20$$

So, 20 is the 9th term in the sequence.

Arithmetic Sequences as Functions

You earn $5 each week for allowance. We can use this situation to represent arithmetic sequences as functions.

First, let's make a table for several weeks.

1st week	$5	$5	
2nd week	$5 + 1(5)	$10	
3rd week	$5 + 2(5)	$15	
4th week	$5 + 3(5)	$20	

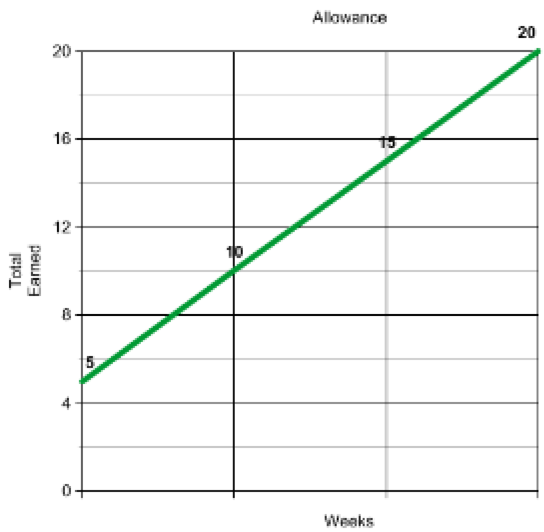
This is called **tabular representation.** This is one of many ways we can represent this situation mathematically.

We can see a function being created in the second column.
$f(x) = 5 + x(5)$ or rewritten in more of a standard form as:

$$f(x) = 5x + 5$$

This is called **<u>function representation</u>**, another way we can represent this situation mathematically.

We can also graph the situation with the weeks being our x axis and money being the y axis. This is called **<u>graphic representation</u>**.



In the allowance situation, we only learned about 4 weeks. Our <u>domain</u> are all of the x values (time/weeks) [1, 2, 3, 4]. Our <u>range</u> is all of the y values (allowance totals) {5, 10, 15, 20].

Now we need to discuss **<u>limits</u>**. In linear equations, lines seems to go on forever. When given just an equation, the line DOES go on forever. But in real world situations, there are limits, or things that stop the line. What might be the limits in this situation?

1. Time has to start somewhere, so one limit is at 0 weeks you have made 0 money.
2. At some point in your life your allowance will either change, or end….your parents won't give you allowance ALL of your life, and even if they do, your parents will eventually die, thus not allowing them to give you allowance. This is another limit.

Finally, we must recognize a **<u>parent function.</u>** This is the simplest form of a function. In this case, the parent function of all linear functions is:

$$\boldsymbol{f(x) = x}$$

This will always look like this: