Alg. Lesson 9-8 Geometric Sequences as Exponential Functions

OAS: A1.A.3.6 Recognize that geometric sequences are exponential using equations, tables, graphs and verbal descriptions. Given the formula f(x) = a (r)^x find the next term and define the meaning of a and r within the context of the problem.
A1.F.2.1 Distinguish between linear and nonlinear (including exponential) functions arising from real-world and mathematical situations that are represented in tables, graphs, and equations. Understand that linear functions grow by equal intervals and that exponential functions grow by equal factors over equal intervals.
A1.F.3.1 Identify and generate equivalent representations of linear equations, graphs, tables, and real-world situations.
A1.F.3.2 Use function notation; evaluate a function, including nonlinear, at a given point in its domain algebraically and graphically. Interpret the results in terms of real-world and mathematical problems.

<u>Recognizing Geometric Sequences</u>
A sequence is simply a pattern of numbers in a specific order. One person sends five emails. Each of those five people send five emails each, then 25 emails are generated. If each of those twenty five people send five emails each, then 125 emails are generated. The sequence of emails generated is 5, 25, 125… This is a geometric sequence.

The first term must be non-zero. Each term after the first is multiplied by the previous term on a non-zero constant "r", which is called the <u>common ratio</u>. This can be found by diving any term by its previous term.

Ex) 5, 25, 125
Divide 25 by its previous term $\frac{25}{5} = 5$ and 125 by its previous term: $\frac{125}{25} = 5$, so 5 is the common ratio.

Ex) Determine whether each sequence is arithmetic, geometric, or neither.
256, 128, 64, 32…
Geometric--$\frac{128}{256} = \frac{1}{2}$, $\frac{64}{128} = \frac{1}{2}$, $\frac{32}{64} = \frac{1}{2}$
Arithmetic—$256 - 128 = 128$, $128 - 64 = 64$, $64 - 32 = 32$

Since the geometric way is constant and the arithmetic is not, it has to be a geometric sequence and not an arithmetic sequence.

Find the next term

Once a common ratio is known, more terms of a sequence can be found.
Ex)1, -4, 16, -64…   Find the next three terms in the geometric sequence.

First, we need to find the common ratio.

Find the common ratio.

$$1 \qquad -4 \qquad 16 \qquad -64$$

$$\frac{-4}{1} = -4 \qquad \frac{16}{-4} = -4 \qquad \frac{-64}{16} = -4$$

Then, we multiply the last term by the common ratio to find the next term: (-64)(-4) so the fifth term is 256. We do this again to find the sixth and seventh terms: (256)(-4)= -1024 and (-1024)(-4) = 4096

So the next three terms are 256, -1024, and 4096.

Find the "n"th term

The "n"th term, just simply means a number in the sequence that we don't know where it sits in the sequence.

For example: in this situation, "a" stands for the terms, and "r" stands for the common ratio.

First term--8	8	A_1	
Second Term--64	8(8)	A_2	$A_1(r)$
Third Term--512	$8(8^2)$	A_3	$A_1(r^2)$
Fourth Term--4096	$8(8^3)$	A_4	$A_1(r^3)$
N th Term-- ??	$8(8^{n-1})$	A_n	$A_1(r^{n-1})$

This also shows the equation for the nth term of a sequence: $a_n = a_1 r^{n-1}$.

Where a_1 is the first term in the sequence and r is the common ratio in the terms.

Ex) What is the 9^{th} term of the sequence?   -6, 12, -24, 48

Use the formula: $a_n = a_1(r^{n-1})$
A_1 is the first term in the sequence, which in this case is -6.
N is the nth term. In this problem, they are asking for the 9^{th} term, so n = 9.

R is the common ratio. In this sequence, the common ratio is $\frac{12}{-6} = -2$.

$$a_n = -6(-2^{9-1})$$

We use order of operations to solve:

$a_n = -6((-2)^{9-1})$

$$a_n = -6(-2)^8$$

$$a_n = -6(256)$$

$$a_n = -1536$$

So, -1536 is the 9th term in the sequence.

Geometric Sequences as Functions

If you drop a ball from 16 feet up, it will only bounce 70% of the height each previous bounce.

First, let's make a table for each bounce.

Original drop	16 feet	16 feet	
1st bounce	16(.7)	11.2 feet	
2nd bounce	$16(.7)^2$	7.84 feet	
3rd bounce	$16(.7)^3$	5.488 feet	

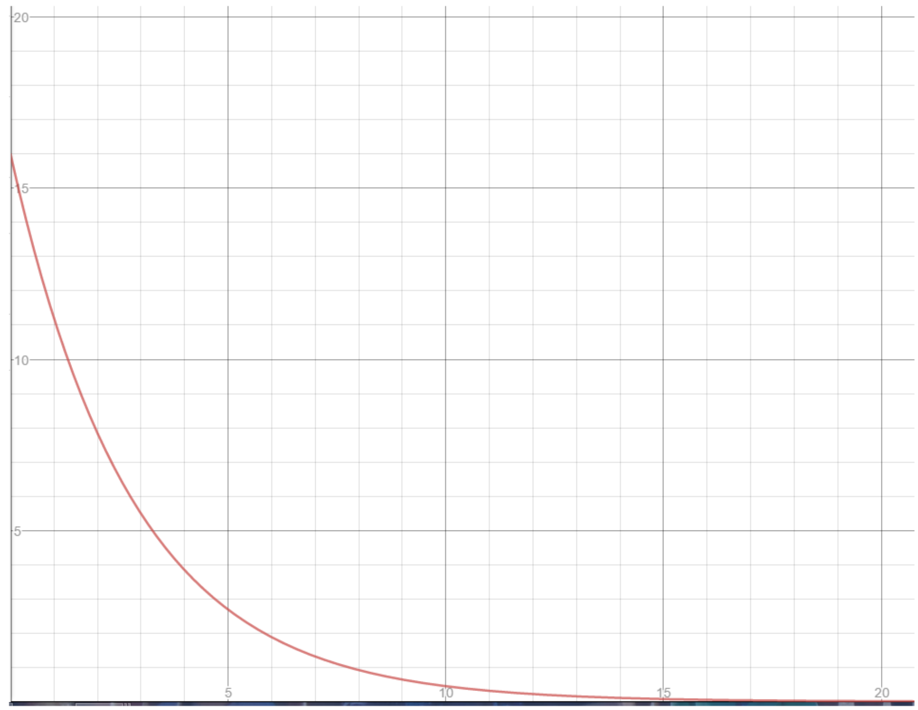
This is called **tabular representation.** This is one of many ways we can represent this situation mathematically.

We can see a function being created in the second column.

$$f(x) = 16(.7)^{x-1}$$

This is called **function representation**, another way we can represent this situation mathematically.

We can also graph the situation with the bounces being our x axis and feet being the y axis. This is called **graphic representation**.



In the ball situation, we only learned about 4 sets of bounces. Our domain are all of the x values (bounces) [1, 2, 3, 4]. Our range is all of the y values (height) {16, 11.2, 7.84, 5.488].

Now we need to discuss **limits**. When graphing equations, lines seems to go on forever. When given just an equation, the line DOES go on forever. But in real world situations, there are limits, or things that stop the line. What might be the limits in this situation?

1. The ball has to start at 16 feet. It cannot be higher than that.
2. At some point the ball will stop bouncing and start rolling.