Geometry Unit 0 Review: Lesson 0-9
Square Roots and Simplifying Radicals

Goal: Evaluate square roots and simplify radical expressions.

OAS: PA.N.1.5 and A1.N.1.1 and 1.2

Vocabulary:
Radicand: the expression INSIDE the radical sign
Radical: the radicand and the radical sign—sometimes known as a square root.
Radical Sign: $\sqrt{}$
Product Property: If a and b ≥ 0, $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$
Quotient Property: If a and b ≥ 0, $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$
Conjugate: Binomials in the form of: $p\sqrt{x} + r\sqrt{t}$ and $p\sqrt{x} - r\sqrt{t}$

Radical expressions contain a radical, such as a square root. The expression under the radical sign is called a **radicand.** We will be simplifying radical expressions….to be simplified they have to meet three criteria:

1. No radicands have perfect squares factors other than 1.

2. No radicands contain fractions.

3. No **radicals** appear in the denominator.

Product property of square roots:

For any non-negative real numbers a and b, the square root of ab is equal to the square root of a times the square root of b. $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$ Ex) $\sqrt{36} = \sqrt{4 \cdot 9} = \sqrt{4} \cdot \sqrt{9} = 2 \cdot 3 = 6$

Simplify Square Roots:

Ex) Simplify $\sqrt{80}$

Step 1: Look for perfect squares in 80. You can do this by math fact knowledge, or you can use prime factorization.

Ex) $\sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 5}$

$\sqrt{4} \cdot \quad \sqrt{4} \cdot \quad \sqrt{5}$

Step 2: Simplify perfect squares and leave imperfect squares as radicands.

Ex) Simplify $\sqrt{80}$

$\sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 5}$

$\sqrt{4} \cdot \quad \sqrt{4} \cdot \quad \sqrt{5}$

$\sqrt{4} \cdot \quad \sqrt{4} \cdot \quad \sqrt{5} = 2 \cdot 2 \cdot \sqrt{5} = 4\sqrt{5}$

Multiplying with Square Roots

To multiply square roots, radicands do NOT have to be the same.

Ex) Multiply: $\sqrt{2} \cdot \sqrt{14}$

Step 1: Multiply together: $\sqrt{2} \cdot \sqrt{14} = \sqrt{28}$

Step 2: Factor out perfect squares: $\sqrt{2 \cdot 2 \cdot 7} = 2\sqrt{7}$

Step 3: check to make sure you have completely factored out all the perfect squares.

Ex) Multiply: $3\sqrt{2} \cdot 2\sqrt{6}$

Step 1: Use Associative property to group the coefficients and the radicals

$(3 \cdot 2)(\sqrt{2} \cdot \sqrt{6})$

Step 2: Multiply together: $6\sqrt{12}$

Step 3: Factor out perfect squares: $6\sqrt{2 \cdot 2 \cdot 3} = 6 \cdot 2\sqrt{3}$

Step 4: Multiply all coefficients: $12\sqrt{3}$

Rationalizing the denominator:

Quotient property of Square roots: For any real numbers a and b, where a $\geq$ 0 and b $\geq$ 0, the square root of $\frac{a}{b}$ is equal to the square root of a divided by the square root of b.

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

Ex) Simplify $\sqrt{\frac{35}{15}}$

Remember that we cannot have a radical in the denominator and we cannot have a fraction as a radicand. The square root of 15 is not a perfect square, so we need to simplify this.

Step 1: Simplify the fraction: $\sqrt{\frac{35}{15}} = \sqrt{\frac{7}{3}}$

Step 2: Use the quotient property of square roots: $\sqrt{\frac{7}{3}} = \frac{\sqrt{7}}{\sqrt{3}}$

Step 3: To get the radical out of the denominator, we use the fraction rule for making equivalent fractions that "whatever I do to the bottom I do to the top"

$$\frac{\sqrt{7}}{\sqrt{3}} \frac{\cdot\sqrt{3}}{\cdot\sqrt{3}} = \frac{\sqrt{21}}{\sqrt{9}}$$

Step 4: Simplify any square roots that you can simplify. $\frac{\sqrt{21}}{\sqrt{9}} = \frac{\sqrt{21}}{3}$ $(\sqrt{21} = \sqrt{3 \cdot 7}\ does\ not\ simplify)$

Step 5: Simplify your fraction if you can (coefficients here are $\frac{1}{3}$ which doesn't simplify)

Using Conjugates to simplify radicals

As we have previously learned in Alg. 1—differences of squares look very much like our **conjugates**: $(p\sqrt{x})^2 - (r\sqrt{t})^2 = (p\sqrt{x} + r\sqrt{t})(p\sqrt{x} - r\sqrt{t})$. We will use this idea to create a square that then we can add a square root to and eliminate some radicals.

Ex) Simplify $\frac{4}{5-2\sqrt{3}}$

Step 1: Use a conjugate to eliminate radicals in the denominator.

Ex) $\frac{4}{5-2\sqrt{3}} \cdot \frac{5+2\sqrt{3}}{5+2\sqrt{3}}$ We know that anything over itself = 1, so we are essentially just multiplying by one here, which won't change the problem's value.

Step 2: We know that $(a-b)(a+b) = a^2 - b^2$. We use this rule to simplify the radical.

Ex) $\frac{4}{5-2\sqrt{3}} \cdot \frac{5+2\sqrt{3}}{5+2\sqrt{3}} = \frac{4(5+2\sqrt{3})}{5^2-(2\sqrt{3})^2}$

Step 3: Simplify by using distributive property and product property of square roots.

Ex) $\frac{4(5+2\sqrt{3})}{5^2-(2\sqrt{3})^2} = \frac{20+8\sqrt{3}}{25-4(3)} = \frac{20+8\sqrt{3}}{13}$

Step 4: Check to make sure you don't have any fractions that will simplify. In this example there aren't any.