Geometry Unit 2: Lesson 2-8 Proving Angle Relationships

Goals: --Write proofs involving supplementary angles and complementary angles.
--Write proofs involving congruent and right angles.
--Use deductive reasoning to prove a statement.

OAS: G.RL.1.1 Understand the use of undefined terms, definitions, postulates, and theorems in logical arguments/proofs.

Vocabulary: (none)

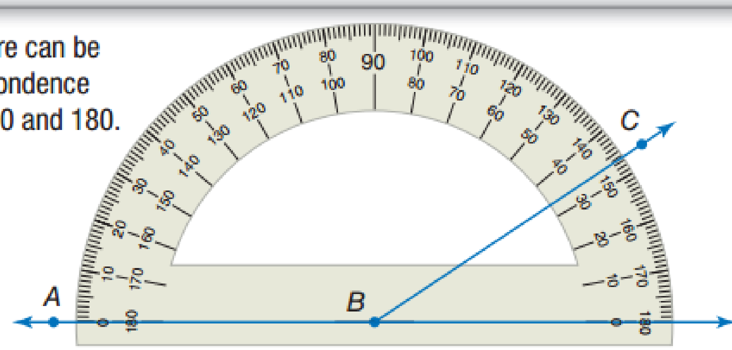
Complementary and Supplementary Angles

The Protractor Postulate shows the relationship between angle measurements and real numbers.

Postulate 2.10 Protractor Postulate

Words Given any angle, the measure can be put into one-to-one correspondence with real numbers between 0 and 180.

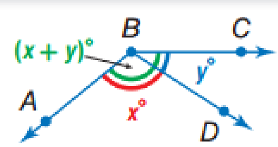
Example If $\overrightarrow{BA}$ is placed along the protractor at 0°, then the measure of $\angle ABC$ corresponds to a positive real number.



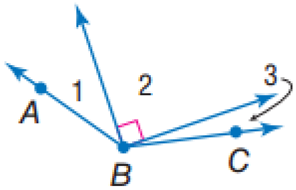
Angle Addition Postulate: When you add two angles together, they will create a third, larger angle.

Postulate 2.11 Angle Addition Postulate

D is in the interior of $\angle ABC$ if and only if $m\angle ABD + m\angle DBC = m\angle ABC$.



Use the angle addition postulate



Ex)

If the m/_1 = 23 and m/_ABC = 131, find the measure of /_3. Justify each step.

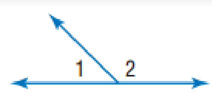
m/_1= 23 and m/__ABC= 131	given
m/__1 + m/_2 + m/_C = m/_ABC	Angle addition postulate
23 + 90 + m/__3 = 131	Substitution (m/_2 = 90)
113 + m/__3 = 131	Substitution
113 – 113 + m/__3 = 131 - 113	Subtraction property of Equality
m/_3 = 18	Substitution

The Angle Addition Postulate can be used with other angle relationships to provide additional theorems relating to angles.

Theorems

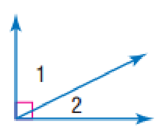
2.3 Supplement Theorem If two angles form a linear pair, then they are supplementary angles.

Example $m\angle 1 + m\angle 2 = 180$



2.4 Complement Theorem If the noncommon sides of two adjacent angles form a right angle, then the angles are complementary angles.

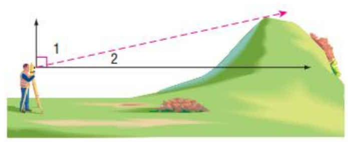
Example $m\angle 1 + m\angle 2 = 90$



Using a supplement or complement

Real-World Example 2 Use Supplement or Complement

SURVEYING **Using a transit, a surveyor sights the top of a hill and records an angle measure of about 73°. What is the measure of the angle the top of the hill makes with the horizon? Justify each step.**



If there is no picture, it is HIGHLY suggested that you draw one.

m/_1 + m/_2 = 90	complement theorem
73 + m/__2 = 90	substitution (m/__1 = 73)
73 – 73 + m/__2 = 90 – 73	subtraction property of equality
m/__2 = 17	substitution

Congruent Angles

**Remember that angles can be congruent and measures of angles can be equal, but not vice versa.

Congruence of segments and the equality of their measurements holds true for the congruence of angles and the equality of their measures.

Theorem 2.5 Properties of Angle Congruence

Reflexive Property of Congruence

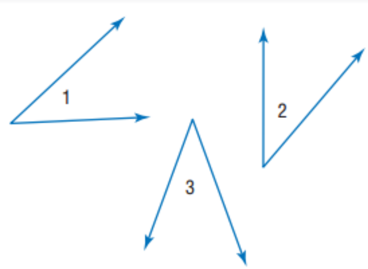
$\angle 1 \cong \angle 1$

Symmetric Property of Congruence

If $\angle 1 \cong \angle 2$, then $\angle 2 \cong \angle 1$.

Transitive Property of Congruence

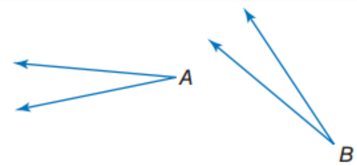
If $\angle 1 \cong \angle 2$ and $\angle 2 \cong \angle 3$, then $\angle 1 \cong \angle 3$.



Proof Symmetric Property of Congruence

Given: $\angle A \cong \angle B$

Prove: $\angle B \cong \angle A$



Paragraph Proof:

We are given $\angle A \cong \angle B$. By the definition of congruent angles, $m\angle A = m\angle B$. Using the Symmetric Property of Equality, $m\angle B = m\angle A$. Thus, $\angle B \cong \angle A$ by the definition of congruent angles.

ReadingMath

Abbreviations and Symbols

The notation $\measuredangle$ means *angles*.

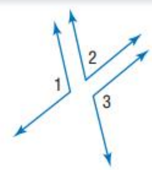
Theorems

2.6 Congruent Supplements Theorem

Angles supplementary to the same angle or to congruent angles are congruent.

Abbreviation $\measuredangle$ suppl. to same $\angle$ or $\cong$ $\measuredangle$ are $\cong$.

Example If $m\angle 1 + m\angle 2 = 180$ and $m\angle 2 + m\angle 3 = 180$, then $\angle 1 \cong \angle 3$.

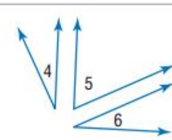


2.7 Congruent Complements Theorem

Angles complementary to the same angle or to congruent angles are congruent.

Abbreviation $\measuredangle$ compl. to same $\angle$ or $\cong$ $\measuredangle$ are $\cong$.

Example: If $m\angle 4 + m\angle 5 = 90$ and $m\angle 5 + m\angle 6 = 90$, then $\angle 4 \cong \angle 6$.



****ERROR ALERT—Make sure you read the problem carefully to make sure you are finding what the question is asking. Some questions may ask you to find an angle measurement, but you have to solve for x to find it. Make sure you do not stop at just finding x!!!**

Proof One Case of the Congruent Supplements Theorem

Given: $\angle 1$ and $\angle 3$ are supplementary.
$\angle 2$ and $\angle 3$ are supplementary.

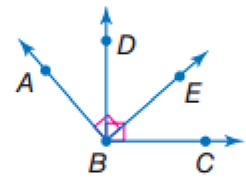
Prove: $\angle 1 \cong \angle 2$

Proof:

Statements	Reasons
1. $\angle 1$ and $\angle 3$ are supplementary. $\angle 2$ and $\angle 3$ are supplementary.	**1.** Given
2. $m\angle 1 + m\angle 3 = 180$; $m\angle 2 + m\angle 3 = 180$	**2.** Definition of supplementary angles
3. $m\angle 1 + m\angle 3 = m\angle 2 + m\angle 3$	**3.** Substitution
4. $m\angle 3 = m\angle 3$	**4.** Reflexive Property
5. $m\angle 1 = m\angle 2$	**5.** Subtraction Property
6. $\angle 1 \cong \angle 2$	**6.** Definition of congruent angles

GuidedPractice

3. In the figure, $\angle ABE$ and $\angle DBC$ are right angles. Prove that $\angle ABD \cong \angle EBC$.



Ex)

Given: $\angle ABE \cong \angle DBC$ Prove: $\angle ABD \cong \angle EBC$

$\angle ABE \cong \angle DBC$	given
$m\angle ABE = m\angle DBC$	Definition of congruence
$m\angle ABE = 90; m\angle DBC = 90$	Substitution (right angles are given)
$\therefore \angle ABD + \angle DBE = 90; \angle DBE + \angle EBC = 90$	Complementary angles
$m\angle ABD = m\angle EBC$	Congruence complements theorem
$\angle ABD \cong \angle EBC$	Definition of congruence

<u>Vertical Angles</u>

If two angles are vertical, then they are congruent. A symbol you may see is: $\measuredangle$s

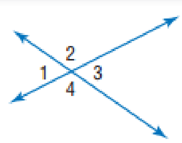
This means "angles (plural). Ex) Vert. $\measuredangle$s are ≅ . (Vertical angles are congruent)

Theorem 2.8 Vertical Angles Theorem

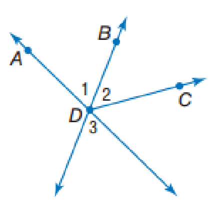
If two angles are vertical angles, then they are congruent.

Abbreviation Vert. $\measuredangle$s are ≅.

Example $\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$



Ex) Prove that if $\overrightarrow{DB}$ bisects $\angle ADC$ then $\angle 2 \cong \angle 3$.



Prove: $\angle 2 \cong \angle 3$ Given: $\overrightarrow{DB}$ bisects $\angle ADC$

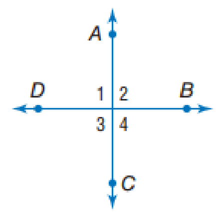
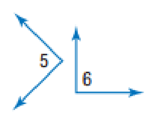
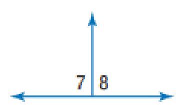
$\overrightarrow{DB}$ bisects $\angle ADC$	given
$\angle 1 \cong \angle 2$	Definition of angle bisector

∠1 ≅ ∠3	Vertical angle theorem
∠2 ≅ ∠3	Transitive property of congruence

<u>Right Angle Theorems</u>

These can be used to prove right angles.

Theorems Right Angle Theorems

Theorem	Example
2.9 Perpendicular lines intersect to form four right angles. **Example** If $\overleftrightarrow{AC} \perp \overleftrightarrow{DB}$, then ∠1, ∠2, ∠3, and ∠4 are rt. ∡.	
2.10 All right angles are congruent. **Example** If ∠1, ∠2, ∠3, and ∠4 are rt. ∡, then ∠1 ≅ ∠2 ≅ ∠3 ≅ ∠4.	
2.11 Perpendicular lines form congruent adjacent angles. **Example** If $\overleftrightarrow{AC} \perp \overleftrightarrow{DB}$, then ∠1 ≅ ∠2, ∠2 ≅ ∠4, ∠3 ≅ ∠4, and ∠1 ≅ ∠3.	
2.12 If two angles are congruent and supplementary, then each angle is a right angle. **Example** If ∠5 ≅ ∠6 and ∠5 is suppl. to ∠6, then ∠5 and ∠6 are rt. ∡.	
2.13 If two congruent angles form a linear pair, then they are right angles. **Example** If ∠7 and ∠8 form a linear pair, then ∠7 and ∠8 are rt. ∡.	

Recall that ⊥ means "is perpendicular to"

Ex) Prove Theorem 2.9 that Perpendicular lines intersect to form right angles.

Prove: ∠1, ∠2, ∠3, ∠4 are right angles Given: $\overleftrightarrow{AC} \perp \overleftrightarrow{DB}$

$\overleftrightarrow{AC} \perp \overleftrightarrow{DB}$	given
m∠1 + m∠2 are a linear pair and = 180	Supplement theorem(T2.3)
m∠2 + m∠4 are a linear pair and = 180	Supplement theorem(T2.3)
m∠3 + m∠4 are a linear pair and = 180	Supplement theorem(T2.3)
∠1 ≅ ∠4	Congruent supplement theorem(T2.6) and Vertical angles theorem(T2.8)
∠2 ≅ ∠3	Congruent supplement theorem(T2.6) and vertical angles theorem(T2.8)
∠1 ≅ ∠2 and ∠3 ≅ ∠4	Perpendicular lines form congruent adjacent angles (T2.11)

$\angle 1$, $\angle 2$, $\angle 3$, $\angle 4$ are right angles	If two angles are cong. And supp., then each angle is a right angle. (T2.12)